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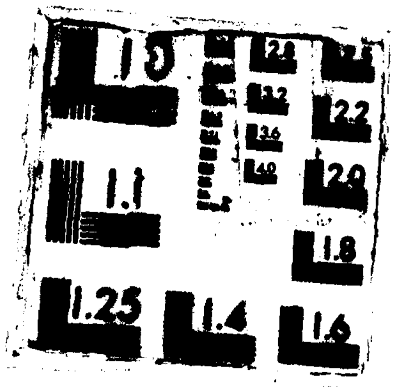
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COMMENTS ON THE DEVELOPMENT OF COMPUTATIONAL MATHEMATICS IN CZECHOSLOVAKIA AND IN THE USSR

I. Babuška

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Abstract

This document presents ~~The talk is~~ an invited lecture at the ACM Conference on the History of Scientific and Numeric Computations, May 13-15, 1987, Princeton, New Jersey. It present some basic subjective observations about the history of numerical methods in Czechoslovakia and USSR up to the mid 60s. (Keywords: freezing; elasticity; error control; variational methods; finite difference theory; differential equations).

COMMENTS ON THE DEVELOPMENT OF COMPUTATIONAL MATHEMATICS IN CZECHOSLOVAKIA AND IN THE USSR*

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Institute for Physical Science and Technology, University of Maryland

At the request of the organizing committee, I would like to share some of my observations and remembrances about the development of computational mathematics in Czechoslovakia and the USSR. My observations will be very subjective and broad in scope.

A. The Development in Czechoslovakia

1. The development until 1918

A very essential milestone in the development of science in Central Europe was the foundation of the Charles University in Prague in 1348. To my knowledge, the first mathematical text at this University was likely Algorismus Prosaycus by Krištan, from Prachatice (in Czechoslovakia) written in 1400. This text concentrates on arithmetic and so I see it as the first text on computational mathematics in Central Europe.

Many outstanding mathematicians interested in computations were, directly or indirectly, for shorter or longer periods, associated with the Charles University. Let me mention the astronomers T. Brahe (1546-1601), J. Kepler (1571-1630) and J. Bürgi (1552-1632), among others. The silver mining in Bohemia (the major mining place in Europe at this time) and the construction of a system of ponds in Southern Bohemia required significant effort and high accuracy in geodesic measurements and computations. This, together with the need of astronomy, contributed to the development of computational mathematics. Computational methods of Brahe (how to multiply numbers by additions with help of tables of sin and cos) together with the logarithmic (tables of Napier, Kepler, Bürgi), and the development of a mechanical computer by Schickart, from Tübingen in Germany (1592-1632), based at Kepler's inspiration, lead to new developments of computational mathematics. The Algebra by Bürgi was edited by Kepler, especially because it contributed to the computational techniques. Many other important developments happened in Prague, especially in connection with the University; nevertheless, I will not go into details except to emphasize that this progress was very closely related to the development of applied mathematics.

2. The period 1918-1945

After World War I, Czechoslovakia was established as a democratic republic. The development of computational mathematics was closely related to applications especially in engineering. Let me mention as an example the fields with which I am familiar, the structural mechanics, elasticity, strength of material. One outstanding scientist in this direction was Z. Bažant, professor of the Technical University in Prague. Traditionally, computational methods for the analysis of frame constructions were of great interest. Essentially, these techniques were related to the direct and iterative methods for solving systems of linear algebraic equations. These usually sophisticated methods were based rather on physical and engineering intuition than on mathematical theories, because at this time maximal simplification was needed for any computation. Some of these methods could be described today as the splitting method, block iterations, some as the method of dimensional reduction, etc.

Approximate methods for analysis of plates and shells based, for example, on Fourier method, series method, etc., were typical for solving partial differential equations. Various solution methods had the character of finite differences derived on physical grounds by "spring analyses." Let us mention that Cauchy's spring model of an elastic medium can be interpreted as finite difference scheme for Lamé-Navier equations with Poisson ratio $\nu = 1/3$. Various methods for solving nonlinear problems, eigenvalue problems, etc., were developed in connection with buckling and stability considerations in general. In mechanical engineering various methods were developed in connection with vibration problems, etc.

The first mathematical book [1, 1934] written in 1934 by two professors of mathematics at the Technical University in Prague became a widely used text. This book covered essentials of numerical analysis in a relatively accurate and detailed manner. Although this book did not brake new grounds or introduced new approaches, it became a major source of education in computational mathematics and in computational research in en-

*Invited lecture at ACM Conference on the History of Scientific and Numeric Computation, May 13-15, 1987 Princeton, New Jersey.

gineering applications in Czechoslovakia.

Czechoslovakia was a highly developed industrial country. The Škoda enterprises, an industrial concern, supported a theoretical department which was heavily involved in computations. Thanks to that, Czechoslovakia had a broad and firm tradition in applied mathematics and through it in computational methods.

It is interesting to compare the scientific situation in Czechoslovakia and Poland. Without any doubts, Poland was a superpower in pure mathematics during this time; it was in the absolute forefront of the world research in developing such mathematical fields as Functional Analysis, Real Analysis, Topology, etc. On the other hand, in my opinion, the level of applied mathematics was higher in Czechoslovakia than in Poland.

In the Fall of 1938, Czechoslovakia was crippled by the Munich treaty; on March 15, 1939, Hitler occupied Bohemia and Moravia, the industrial western part of Czechoslovakia, and created a puppet state of Slovakia from the eastern part of Czechoslovakia. In other words, Czechoslovakia ceased to exist.

On November 17, 1939, Hitler closed all universities to prevent the higher education of the Czech population. Universities were closed until the end of the war and the collapse of Hitler's Germany. This, of course, had a profound effect in the development of science in general, and mathematics in particular. Although there were underground seminars and some mathematical work and some more elementary publications were somehow published, an entire generation of scientists (6-8 years period) was lost. (Some effects of this will be discussed in the following sections.)

3. The early post war period. Period of basic education

Almost immediately after the end of the war, the Universities were opened and maximal efforts started to fill the gap created by the closing of the schools for six years. Shortened studies were designed to fill the gap as quickly as possible. Basic lectures were given in theaters for 1500-2000 students. This emergency education had surprisingly good effects because of the high motivation of the students and teachers. In three to four years the major part of the educational gap was closed, especially in the education of engineers, teachers, medical personnel, etc., but could not and was not completed in the field of science and in the education of scientists.

In February 1948, the Communist party took over the government. The pattern of Soviet organization was applied in Czechoslovakia including scientific education and research. Already in 1949 the institution of "Aspirants" was established. "Aspirantura" was an organization for graduate studies in and outside the universities. Aspirants were awarded fellowships. Almost at the same time, preparations for the foundation of the Academy of Sciences (Soviet style) was started.

In mathematics the major responsibility for the education of aspirants was given to E. Čech, professor at Charles University, a well known topologist. He gathered about a dozen of the best and most promising young students, graduates from the universities, and led their scientific education. Let me mention a few names from this group which became well known in mathematics in and outside of Czechoslovakia. I. Babuška (Numerical and Applied Mathematics), M. Fiedler (Theory of Matrices), J. Kurzweil (Theory of Ordinary Differential Equations), V. Pták (Functional Analysis), O. Vejvoda (Differential Equations), M. Zlamal (Finite Element Method). Under the leadership of E. Čech, the best Czechoslovak mathematicians participated in this program. I would like to mention especially V. Knichal, V. Kořinek, professors at Charles University in Prague, F. Vyčichlo, Professor at Technical University, O. Boruvka, Professor at the University in Brno. This group of students and their teachers were a congenial, dedicated group of the highest quality. I have not seen afterwards anywhere in the world such a congenial group of students and teachers.

Professor E. Čech, although a pure mathematician with basic interest in topology and geometry, had very broad views which he imposed on the group together with his dedication, hard work and interest in every aspirant (student). E. Čech insisted that all of his "aspirants" became familiar with numerical methods. To this end, he obtained from the Soviet Union some old copies of the book of Kantorovich Krylov [2, 1936], which was well known in the Soviet Union and was translated later in the West. Because the copying machine did not exist at that time in Czechoslovakia with the exception of the ditto sheet machine, E. Čech translated and dictated it to his secretary, so that the entire book was typed and by ditto technology given to his aspirants. This and similar Čech's acts were typical of his dedication. Nevertheless, it is necessary to say that Prof. E. Čech was a highly demanding person, completely "obsessed" by mathematics (in the best sense of the word) who permanently challenged his students individually and as a group almost in a dictatorial fashion. In retrospect, one has to admire more and more his mathematics, dedication, wisdom and what he gave to "his" youngsters (with or without their consent).

E. Čech also insisted that the aspirants will get basic education in computer technology and its use. He arranged for lectures by Prof. A. Svoboda. A. Svoboda worked in the field of electronics in the United States during World War II. He returned to Czechoslovakia in 1946 and went back to the United States in 1966. A. Svoboda was the leader in the development of computers in Czechoslovakia. Under his leadership, a design and implementation of a unique relay computer was made (tubes were not available at this time). Svoboda's machine called SAPO was a triplet machine with three arithmetic units which after every operation (made simultaneously) "voted" and the majority vote was used as the answer. The programming was a 5 address system. The computer SAPO had many unique features. Unfortunately it was completed when the next generation (tubes) was already in full swing.

During this period, work seminars were routine. Teachers, as well as students, were involved in these seminars. I remember, for example, the work in a paper by Goldstine, Neumann [3, 1947] which convinced us that there was no hope that elimination method could and would be used in the future for matrices larger than 100 (what a wrong conclusion!) Another paper having big impact was the one by Courant, Friedrich and Levy [4, 1927] which was analyzed in every detail; E.

Čech and others made many comments related to the connection to other fields of mathematics.

E. Čech, V. Kořínek, V. Knichal and F. Vyčichlo were able to grow a new generation of very active mathematicians and fill the gap of the closing of universities by Hitler in a relatively short period of time. (Let us mention that also with an extraordinary effort, it needed eight to ten years to overcome the basic effects of this closing.)

4. Building the Mathematical Institute of the Czechoslovak Academy of Science

In the early fifties, the Mathematical Institute of the Czechoslovak Academy was established. E. Čech, V. Knichal, J. Novák, F. Vyčichlo, together with some of the previous aspirants, played a prominent role in leading the Institute. New research groups were built and another generation of young researchers educated.

In the field of Applied and Numerical Mathematics and Partial Differential Equations, I. Babuška and K. Rektorys* became very active in collaboration with Prof. F. Vyčichlo.

The main emphasis in this direction of applied mathematics was mechanics of solids and partial differential equations, especially of elliptic type. The main direction was the relation between modern exact mathematics and applications with emphasis on constructive approaches which could be used for concrete solution of problems. One of the result of this effort was the book [6, 1953]. The basis of this book was the theory of analytic functions of complex variables in the spirit of the Muschelishvili theory. This philosophy of the honest mathematics in application later led to the book [5, 1966] by K. Rektorys and coworkers.

The above philosophy in its purest form, and influenced by Bourbaki, led to some effort (e.g. by V. Knichal and others) to create an axiomatic-precise system of applied mathematics. This effort did not accomplish too much.

The early post-war period (I call it 'period of education') ended roughly in 1954 when the Mathematical Institute was firmly established.

5. The Project Orlik

The project Orlik was an important milestone in the development of computational and applied mathematics in Czechoslovakia. This project was mentioned as the one of the principal achievements of the Czechoslovak Academy of Sciences on the occasion of the celebration of 30 years of its foundation, and in the publication [8, 1986] on the occasion of forty years of post-war mathematics.

The project Orlik was a large scale computational project (although still performed on desk calculators) which could be characterized as the transition from the precomputer to the computer era in Czechoslovakia, see, e.g. [9, 1986]. This project had a profound impact and was characterized by the principles which after thirty years are still the center of interest in computational and applied mathematics in the United States and elsewhere.

The research project Orlik was related to the proposed building of the largest dam in Czechoslovakia located about 40 m south from Prague on the river Vltava. The dam was of concrete gravitational type, about 400 ft high. The project Orlik was an integrated complex research in mathematics, engineering and material science (cement, concrete). The leader of the mathematical part was I. Babuška, of the engineering part Prof. L. Mejzlík (Professor of Tech. University Brno), and of the technological part, Dr. J. Jirsák. The project was a team work and included a large staff of people working on desk calculators.

The main technical problem was that the concrete releases a significant amount of heat during hardening. Simultaneously, the hardening, which depends strongly on the temperature, changes significantly the material properties, e.g. elasticity module, creep and relaxation properties, etc. This leads to the creation of significant stress state which is "frozen in" during the hardening and later could lead to dangerous and serious cracks. The effects of this type could be controlled by a proper technology of building and of material properties. The large dams in the United States used a cooling system by pipes inserted in the dams. The basic questions of the research were: a) What are the effects of various building procedures on the possible cracks? Is it necessary to use pipe cooling, etc.? Could the cracks, if any, be expected? b) How the properties of the concrete influence the undesired effects of building, later functions of the dam, etc.? Based on the research results, the dam was built without cooling by a relatively quick building schedule in blocks about 12 ft high. The dam behaved as predicted and serves well its purpose. Results of the analysis were presented at the dam world congress in 1958, and were included distinguishly in the congress reporter's report. Some technical conclusions are, e.g. in [10, 1958], [11, 1961].

The essential novelty was the emphasis on integrated approach and the reliability of the conclusions. The reliability aspects were divided in the following groups:

a. reliability of mathematical model,
b. reliability of available input data,
c. reliability of the numerical method and principles of its selections,
d. reliability of the arithmetic computations (round offs) (because minimal number of used digits were essential for computations on desk calculators).

These questions were directly and indirectly addressed in a series of theoretical and engineering papers and reports.

The problem was highly nonlinear and three-dimensional. Because three-dimensional solution was out of the question for obvious reasons, a series of two-dimensional problems were solved and combined approximately into three-dimensional ones by a sort of splitting up approach. Let me explain now some of the problems (in a simplified way).

---

*K. Rektorys is the author of [5] and [7], which are well known in the United States.

1) Thermoproblem with and without cooling. The basic equation which was considered was:

$$(1a)\quad c(u,\delta)\,\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\,a(u,\delta)\,\frac{\partial u}{\partial x} + \frac{\partial}{\partial y}\,a(u,\delta)\,\frac{\partial u}{\partial y} + F(u,\delta)$$

$$(1b)\qquad \frac{d\delta}{dt} = G(u,\delta).$$

Here u is the temperature, F the intensity of the heat created by hydration, δ a fictive time (age) in which the same amount of heat was produced as when the temperature would be fixed (about 70°F). This fictive time characterizes the state of the chemical reaction. The coefficients $c(u,\delta)$ and $a(u,\rho)$ were found so mildly dependent on u,δ that average values were used. The characterization of $F(u,\delta)$ was essential. A special care was devoted to the laboratory experiments. Finally, the above mentioned model, based on a chemical model of hydration, was accepted and a differential equation (1a,b) was designed and used. The data were obtained by the measurement of the heat release in the period $(0,t)$ under constant temperatures and in an adiabatic state. The computation of the increments in F was organized so that the total heat was exactly preserved. This was very essential for the reliability.

The technology of the building consisted in quick production of blocks about 12 ft high with time intervals T in between. The scheme is shown in Figure 1. To simplify the problem, a periodic solution (in time and space) was analyzed. It has been shown that the solution quickly approaches the state $u(t+T,x,y+d) = u(t,x,y)$, $0 \le t \le T$ *and this state was numerically computed* [12, 1960].

The numerical method was essentially the finite difference method with the scheme derived by the cell integration identity principles guaranteeing the balance condition. This technique was close to the technique of Marchuk's identity, elaborated later in [13, 1966].

An essential feature which was introduced much later in the finite element method under the name 'special elements' was used in the computations. In the presence of cooling pipes there was a significant heat sink. Hence, the solution was written in the form

$$u(x,y,t) = v(x,y,t) + w(x,y,t)$$

where $w(x,y,t)$ was the linear solution of a point source (more precisely single circle source) with the intensity $c(t)$ (which was the computed intensity of cooling). Function v was determined by finite difference method as explained above and the hydration heat was included in this term. (For the stationary solution exactly the method of special elements was obtained.)

2) The freezing problem. The building of the dam had to continue during winter when freezing of the concrete in the beginning phase of hardening could create a serious damage. At most the concrete is allowed to freeze for a short time at a depth of 1 - 2 in. The wooden siding for laying the concrete serves also as insulation, and the freezing occurs when the siding is moved in the next building cycle. The main approach was here to solve a stochastic problem for Equation (1) when the boundary conditions are a stochastic function - the outside temperature. The main value and dispersion for the desired information were computed. The theoretical base was described in [14, 1961]. (Let us remark that today a large research project, sponsored by NASA Lewis, solving this problem with stochastic input data is in progress.)

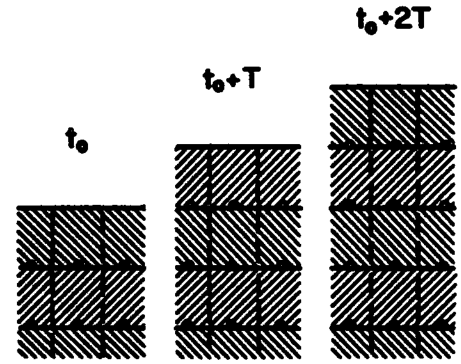

Fig. 1. Schematic state of progressed dam.

3) The elasticity problem. Given the temperature, the thermostresses were computed. The essential problem was the formulation of the problem with respect to material properties including change of elasticity module and creep (relaxation) properties, etc. A rheological model based on a description of the chemical process of hardening was designed and tested in the laboratory.

The numerical solution was based on a series of plane problems in the spirit of splitting up methods. In this phase, J. Nečas contributed significantly to this research. Among others, the theoretical papers [15, 1958] [16, 1959] are directly related to this work. The monograph [17, 1967] by J. Nečas is the only basic monograph which does not avoid unsmooth domains. This monograph and other results of J. Nečas are well known in the West. Various iterative methods were used in the connection of splitting the problem into two dimensional ones. Let us mention one of the, type of Schwarz alternating algorithm. Mathematically, the main generalization used was based on the following functional analytical frame (which is today more or less standard), formulated here in the simplest form:

Let P_1, P_2 be projection operators on the subspaces $S_1, S_2 \subset H$. Then $(P_2P_2)^n$ converges pointwise to the projection onto $S_1 \cap S_2$.

4) Error control. The basic idea of the error control of the numerical method was to interpret the numerical solution as exact solution of a problem with slightly different input data. The mathematical models were verified by computation of some simple laboratory experiments. The round-off error was analyzed in a way close to that explained later in the monograph [13, 1968] by α-sequences. In the project Orlik, a team of researchers was involved. In addition to those already mentioned, I. Babuška, L. Mejzlik, J. Jirsák, E. Vitásek, J. Nečas, other researchers participated, especially K. Rektorys, M. Práger,

F. Vyčichlo. Various publications and reports, which directly or indirectly were related to the project, were published during this time.

6. The research in the optimization of the numerical methods, numerical stability and numerical methods in general

During the sixties (1964, 1967), conferences devoted to numerical mathematics were organized. Emphasis was placed on the questions of optimality of the selection of numerical method and numerical stability. These conferences, which took place in the castle Liblice, were held in a very informal working atmosphere. Leading numerical analysts and mathematicians from east and west participated. Let me mention, among others, N. S. Bachvalov, G. Golub, P. Henrici, G. I. Marchuk, F. Olver, S. L. Sobolev, A. N. Tichonov. These conferences were, in my opinion, the very first meetings in the world concentrating specifically on the questions of optimal selection of the numerical method. The various aspects of optimality, theoretical and computational were discussed. Some ideas and results related to this direction obtained in Czechoslovakia were, for example, presented in [13, 1966].

B. Computational mathematics in USSR

In this section I will make* a few subjective comments about the development of computational mathematics in USSR up to the mid 1950. For a systematic survey, we refer to [18, 1948] and [19, 1959].

The theory of approximate methods has a long tradition. For example, the idea of the Galerkin method was introduced in 1915 in [20, 1915]. The Ritz method was investigated in a series of papers of Krylov and Bogoljubov. See e.g., [21, 1917] [22, 1917], [23, 1927], [24, 1931]. The Galerkin method was investigated by various authors in the pre-war period. The book of Kantorovich and Krylov [2, 1936] is likely the first comprehensive book about the numerical solution of partial differential equations. (After the war this book was translated into many languages.)

The Faddejeva's monography [25, 1950] is likely the first comprehensive book about the methods of linear algebra. (It was later translated into English.) Michlin's work and books (e.g. [26, 1950; 27, 1952] and others) about the variational methods were important contributions to the theory of variational methods and computational approaches.

1. Variational methods

As I have already mentioned, the variational methods were investigated by many authors. The investigations addressed both the Ritz method based on a minimization of a quadratic functional as well as the Galerkin method (sometimes called methods of moments or weighted residuals) with the same or different trial and test spaces. The results related to the applications of a minimization are using the Friedrichs extension of the operator to a selfadjoint one. This direction was utilized by Michlin in many of his papers and books, and Michlin was likely the first who used the term "energy space." An important role played the analysis of the energy space and the question to what Sobolev space (in today's terminology) it is equivalent. For example, in [27, 1952] this question is analyzed for basic problems of the elasticity theory. For the mixed problem (e.g. free friction contact boundary condition) the equivalency was analyzed, e.g. in [28, 1951]. The characterization of the energy space for Poisson problem on an infinite domain was discussed in [29, 1953]. The convergence of the Ritz method in the energy space is then directly related to the best approximation. An effort was made to analyze the convergence in the stronger norms $\|u\| = (Au,Au)^{1/2}$ (see, e.g. [30, 1956]) or weaker norms as $\|\cdot\|_{L_\infty}$ (see, e.g. [31, 1941]. The convergence of the Treftz method was analyzed in detail in [26, 1950].

The Galerkin method and general method of moments (also with different trial and test functions) for integral equations were studied in many papers by Krylov and his coworkers. See, e.g. [23, 1927], [32, 1931]. In applications to differential equation, Petrov [33, 1940] used the different trial and test spaces, and the term Galerkin-Petrov method is used sometimes today. Keldys [34, 1942] applied this method to a nonselfadjoint boundary value problem for ordinary differential equation; this paper very likely was the first one establishing the convergence of the method in general setting when applied to an specific problem. The convergence of the Galerkin method was established by Michlin for the operators of the form $A = A_0 + K$ where A_0 is positive definite selfadjoint operator, and $A_0^{-1}K$ is compact in the norm $(A_0x,x)^{1/2}$. See [35, 1948], [36, 1950], [37, 1957]. In [38, 1948], a general functional analytic scheme of numerical method was discussed by Kantorovich. See e.g., [39, 1960]. The main idea is roughly the following. Let us be interested in $Kx = y$ with $x \in X$, $y \in Y$. Then the numerical method solves essentially $K_hx_h = y_h$ where h is a parameter, $h \to 0$ and $x_h \in \bar{X}$ $y_h \in \bar{Y}$. There is a one to one mapping φ_h of $\bar{X}$ onto $\tilde{X} \in X$ and ψ_h of $\bar{Y}$ onto $\tilde{Y} \subset Y$. Then one would like to achieve that $\varphi^{-1}(x_h)$ is close to the solution of the original problem. For that, one has to essentially achieve that $\varphi_hK - K_h\varphi_h$ is small. In [38, 1948] this approach was applied to a large class of illustrative problems.

Collocation method obviously can also be understood as method of moments and has been treated, e.g. in [39, 1960], in the frame of the above mentioned approach. A method which is very close to the collocation was applied in [40, 1954], [41, 1956] by Vishik. In an abstract form, the Galerkin method and nonlinear problems and a discussion of the approximate method are given by Krasnoselskij in [42, 1954] and in some of his other papers.

*I give here the references to the originals in Russian. Translations of many of these papers and books are now available.

2. Finite difference method

The basic theory of the finite differ nce method especially related to the stability is in the book by Rjabenkij Filippov [43, 1956]. A handbook of finite difference schemes for partial differential equations was written by Panov [44, 1951]. For hyperbolic equations there is a series of results of Ladyzenskaja and her coworkers. See, e.g. [45, 1952], [46, 1952], [47, 1953].

In the case of elliptic equations, early works are given, for example, in [48, 1952], [49, 1947]. For applications of finite difference for parabolic equation, we refer, for example, to the work by Kamynin [50, 1953].

The general eigenvalue treatment by the finite difference method is given, for example, in [51, 1954].

3. Numerical treatment of differential equations

In the previous sections some early works were presented. They played (by the subjective judgement of the author) important roles in the development of the theory of the numerical method. It is interesting to mention that although the theory of variational method was very advanced, the entire direction of the finite element method was for a long time neglected, and the main emphasis was placed on finite difference method. It seems to be characteristic the finite element method was called until recently variational finite difference method.

Finite difference method was later analyzed in the works of Samarskij, Godunov and many others, and many monographs and text books are available today. In these works the emphasis is placed on the theory. The discussions of computational aspects, numerical experimentation, analyses of the performance of the method on benchmark problems are very rare. Very likely this situation is related to the state of the computer technology in USSR. Nevertheless, the computer situation stimulated various special methodologies as splitting up methods, and various "tricky" iterative procedures which were used in scientific computations. In the area of mathematical modeling and scientific computations, important works have been done by G. I. Marchuk and his coworkers in many papers. The first of his books (see [52, 1958] is addressing modeling and computational methods in reactor analysis. It is interesting to mention that the idea of preconditioning--credited to Buljaev--is mentioned there.

I only mentioned very few papers and results; nevertheless, hopefully, they give some illustrative picture of the character of the research in the USSR in the early post-war period.

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